Optimal Generator Policy for Hybrid Fuel UAV under Airspace Noise Restrictions

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Abstract: Here we study an optimal control problem involving energy management of a hybrid-fuel Unmanned Aerial Vehicle (UAV). The planning problem for a hybrid-fuel platform involves determining the path while managing the energy resources, which includes a policy for power modality switching whenever applicable. The hybrid-fuel platform considered here involves a generator and battery pack combined in a series fashion as energy sources on-board a UAV. Also included in the problem are the noise restrictions, which place constraints on generator operation depending on the airspace location. These emulate possible restrictions on UAV noise that occur in military surveillance missions or in urban path planning, where the collective noise of many UAVs, some with combustion engines, may be restricted in certain areas or times of the day. We present a hybrid methodology which starts from an initial path and generator pattern obtained from a mixed integer linear program (MILP) solution. The generator pattern from the discrete solution is then refined in an optimal control framework with an objective to minimize fuel usage, while considering the nonlinear battery and generator dynamics and noise-restriction constraints. Optimal control problem is solved with a nonlinear program solver, IPOPT. Numerical results are presented and analyzed with varying path lengths and scenarios. This work aims to serve as an initial study of this hybrid-fuel UAV problem within an optimal control framework, which can be extended to refinement of both the generator pattern and the trajectory in tandem, while considering vehicle and power dynamics that are often ignored in discrete path planning solutions.

Keywords: UAV, hybrid-fuel, optimal control, path planning, optimization, hybrid methods, nonlinear programming

1. INTRODUCTION

The utilization of multiple fuel sources on a single platform has been implemented extensively in automobiles, with increasing use in Unmanned Aerial Vehicles (UAVs). Fuel hybridization offers similar advantages to UAVs as it does to automobiles. However, the control and autonomy problems for UAVs differ from automobiles and thus hybridization applied to UAVs results in a new set of problems which must be addressed.

Fuel hybridization for UAVs is of utility in Urban Air Mobility (UAM) and UAV Traffic Management (UTM), primarily due to the extended endurance. This extended range broadens the applicability of the fleet of UAVs under consideration to wider variety of missions. For certain hybrid UAVs, the switching of power/energy modalities during the course of a mission could be possible and is advantageous. This results in a new problem of determining the power modality throughout the mission with an objective to maximize energy efficiency or flight time. This can also allow the advantages of each power or energy modality to be maximized throughout the flight and their pitfalls minimized. An overview and survey of hybrid-fuel aircraft is given in Townsend et al. (2020), which also provides characteristics of different fuel sources in the context of UAVs.

We consider here a platform which combines a combustion engine, powered by some liquid fuel, and a battery pack as energy sources. As discussed in Townsend et al. (2020), battery packs provide high power density with poor energy density. Combustion engines have high power and energy density, while being both very heavy and very noisy. The former is a problem for small UAV (sUAV) while the latter is a problem in urban environments or in stealth-

⋆ Approved for public release, distribution unlimited: AFRL-2023-1753

critical missions due to its noise. Here we are motivated by the latter issue, one of excessive noise. In these urban environments or stealth-critical missions, noise production becomes an important consideration in the path planning and power management problems for the UAVs. Running the motors with power from the battery pack while the generator is off allows relatively quieter operation which we consider to be a minimal noise level on the ground. Thus, for certain restricted areas of airspace, the on-board generator cannot be run, and it is free to be turned on or off in unrestricted areas.

This problem of noise-restricted path planning and energy management for UAVs has been considered prior in Scott et al. (2022); Manyam et al. (2022), where a discrete formulation and solution for the problem is presented. Here, we wish to approach the problem from a different direction. Path planning and energy management is often done in a discrete manner on a graph. This approach is so often used as solutions can be obtained quickly and reliably by formulation as a well-studied Shortest Path Problem (SPP) variant, or a formulation close to one of these variants. This allows existing algorithms and techniques to be applied to generate fast solutions. However, this discrete solution must be processed at some point into a smoothed trajectory to be flyable by the UAV. Further, a continuous approach will allow finer management of the UAV's path and power modalities during the mission while also allowing higher-fidelity nonlinear dynamics to be considered rather than a linear approximation.

The problem considered here is using an optimal control framework Kirk (2004); Bryson and Ho (1975) to determine the generator state along a given trajectory to the end of minimal fuel usage. We use a solution from the discrete path and energy management algorithm described in Scott et al. (2022). The generator control returned from the same algorithm is used as an initial guess for the optimizer to solve the optimal control problem (OCP). For a given noise restrictions along the path, the OCP aims to refine the generator control while minimizing the expended fuel. The generator control returned from the discrete pattern is a binary one, specifying the generator only as on/off, however the engine and batteries have their own dynamics and may operate at non-extreme values to the end of higher energy efficiency. Using optimal control allows refinement of the discrete solution while using the discrete solution as an initial guess improves the overall computation time. In this paper, our focus is to compute the generator control that minimizes the fuel consumption for a given path. However, the final aim of this work is to develop a method which computes both the generator control and the trajectory in tandem, which is a considerably harder problem. This extension of the problem is further discussed in Section 6.

The approach in this work begins with the solution to the discrete problem obtained as an initial guess Scott et al. (2022) to be refined within optimal control problem. such that the optimal generator control to minimize fuel usage is find. To find the optimal generator control over a given flight, the direct method of optimal control is leveraged Kelly (2017); Betts (1998, 2010). Rather than using the indirect method of optimal control Kirk (2004); Bryson and Ho (1975) which solves for costates and in-turn provides the optimal control, the direct method performs a search for the optimal control directly. The direct method, as presented in this work, searches for an optimal control using a nonlinear program (NLP) solver that minimizes a given objective cost functional subject to a system of ordinary differential equations, equality, and inequality constraints. The hybrid method (using a discrete solution as an initial guess for an OCP) is illustrated in Fig. 1.

This paper is organized as follows. A discussion of related prior works is given in Section 2. In Section 3 the MILP formulation of the discrete problem, taken from the authors' prior work, is re-presented in brief. The Optimal Control Problem is formulated in Section 4. Numerical results are presented in Section 5. Finally, conclusions are made and future work discussed in Section 6.

2. PRIOR WORK

The path planning and power management of a hybrid-fuel UAV under consideration was discussed and analyzed in Scott et al. (2022); Manyam et al. (2022). Other works concerned with path, trajectory, and power planning of hybrid-fuel UAVs are found in Klesh and Kabamba (2009); Hosseini and Mesbahi (2016), where the problem is solved in an optimal control framework. Hybrid-fuel vehicle control strategies in general have been studied prior, with a main focus on powertrain management and control, a few primary examples being Bumby and Forster (1987); Jalil et al. (1997); Sinoquet et al. (2011).

When formulated as a discrete problem, the hybrid-fuel UAV planning problem is reminiscent of the Resource Constrained Shortest Path Problem (RCSPP). In the RCSPP (Pugliese and Guerriero (2013)), resources are tracked along the path where bounds are placed such that certain paths are infeasible due to resource consumption. Often, fuel or energy is modeled in this fashion. The standard problem does not allow resource regeneration, e.g. a refueling node/arc. A version allowing resource regeneration is known as the RCSPP with Regeneration (RCSPP-R), and has been scarcely studied relative to the main variant, with two recent studies on this problem found in Bolivar et al. (2014) and Smith et al. (2012). For the standard RCSPP-R, replenishment is implicit based on the path taken. That is, nodes and edges are defined a-priori as replenishment events, rather than regeneration being an explicit decision. The only prior work, to the author's knowledge, considering an RCSPP problem where replenishment of resources is decided explicitly is found in the network relay problem in Cabral (2005); Cabral et al. (2007, 2008), as well as the studies directly on hybrid-fuel UAV path planning in Scott et al. (2022); Manyam et al. (2022).

Prior work has considered hybridization of optimal control with other schemes for US Air Force missions Jodeh (2015); Humphreys (2016); Zollars (2018). Jodeh provided solutions from a traveling salesman problem (TSP) prior to using direct orthogonal collocation (DOC) solved by a nonlinear program solver (NLP) for persistent surveillance and data collection Jodeh (2015). Humphreys presented a particle swarm optimization (PSO) to obtain initial tracks for DOC and then leveraged an NLP solver for obtaining optimal strategies for an unmanned aerial system (UAS) to avoid threat regions and support a teammate Humphreys (2016). Zollars used constrained Delaunay triangulation to map a field containing obstacles and then used A* search to provide initial guesses for DOC and NLP solutions to find min-time strategies for navigating through urban environments Zollars (2018).

3. PATH AND GENERATOR PROBLEM FORMULATION IN MILP FRAMEWORK

For completeness, we restate briefly the Mixed Integer Linear Program (MILP) as given in Scott et al. (2022). The problem is posed on a graph with a set of nodes N and edges E. The edges are chosen so as to not intersect predefined keep-out zones; thus the formulation naturally addresses staying out of the predefined keep-out zones. The problem is formulated using set of binary variables x_{ij}'s; x_{ij} is equal to one, if the edge (i,j) is used in the solution, and it is zero otherwise. Another set of binary variable g_{ij}'s indicate if the generator is turned on along edge (i,j). Graph parameters include D_{ij} representing the cost of edge (i,j), C_{ij} for battery state of charge (SOC) usage, Z_{ij} for both energy transfer to the battery and fuel burn by the generator, and G_{ij} for noise restrictions. The battery and fuel state at any node i are tracked by b_i and g_i respectively. The values B_0 and Q_0 are the initial battery and generator states respectively and B_{max} and B_{min} are the maximum and minimum battery charge respectively.

$$\min_x \sum_i \sum_j D_{ij} x_{ij} \tag{1}$$

$$\sum_{j \in N} x_{Sj} = 1 \tag{2}$$

$$\sum_{j \in N} x_{jF} = 1 \tag{3}$$

$$\sum_{j \in N} x_{ij} - \sum_{j \in N} x_{ji} = 0 \qquad \forall i \in N \setminus \{S, F\} \tag{4}$$

$$B_{min} \leq b_j \leq B_{max} \qquad \forall j \in N \setminus S \tag{5}$$

$$b_S = B_0 \tag{6}$$

$$b_j \leq b_i - C_{ij} + Z_{ij} g_{ij} + M(1 - x_{ij}) \qquad \forall (i.j) \in E \tag{7}$$

$$q_j \geq 0 \qquad \forall j \in N \setminus S \tag{8}$$

$$q_S = Q_0 \tag{9}$$

$$q_j \leq q_i - Z_{ij} g_{ij} + M(1 - x_{ij}) \qquad \forall (i.j) \in E \tag{10}$$

$$g_{ij} \leq x_{ij} \qquad \forall (i.j) \in E \tag{11}$$

$$g_{ij} \leq G_{ij} \qquad \forall (i.j) \in E \tag{12}$$

The description all equations are provided in Scott et al. (2022). The above problem seeks to find a path through the graph from the start S to end node F with minimal cost, while satisfying the battery, generator, and noise-restriction constraints. This can be solved in a variety of ways, with the original paper using both a branch-and-bound algorithm and a labeling algorithm which utilizes dynamic programming. As shown in Scott et al. (2022), this problem can be solved on graphs of thousands of nodes. While this can provide fast solutions, it does not account for the vehicle dynamics, and the battery and generator dynamics are simplified to linear relationships. Therefore, post-processing must be done in some manner to get a smooth prediction for the UAV's generator and battery states and trajectory. The MILP above also only allows the generator to be either on (full throttle) or off. However, it may be beneficial to allow the generator throttle to take on a value between off and maximum throttle. To this end, we define an OCP that determines a continuous generator control along the path generated by the discrete solution, to the end of minimal fuel use accounting for the nonlinear power generator and battery system dynamics; the starting solution for the OCP is derived from the MILP solution.

4. OPTIMAL CONTROL

The optimal control problem determines the optimal generator policy (along a given path) that minimizes fuel consumption while satisfying the noise-restrictions. The current objective is to optimize the resources of the platform; the coupled path-planning and resource management problem, in the context of an optimal control framework, is left as future work. This paper focuses solely upon the optimization of the energy resources, where the path followed is provided by a MILP solution. We consider the same platform as in Section 3, where a generator charges a battery pack and the battery pack provides power to the electric-only motors.

A path $\overrightarrow{H}(t) = \{x(t), y(t), z(t)\}$ is given, from initial time t_0 to final time t_f. The power consumption of the motors $T(t)$ is defined over the time interval $[t_0, t_f]$. For the problem studied here, this is assumed to be unchanging as the path is pre-planned and only the generator control is to be considered. The fuel level $g(t)$ and battery state $b(t)$ are given at the initial time as $g(t_0)$ and $b(t_0)$ respectively. The control input $u(t)$ is generator throttle, determining the amount of power produced by the generator to send to the battery. The objective is to minimize fuel usage over the path, while satisfying the noise restrictions. The state space is defined as the fuel level and the battery state of charge $\mathbf{x}(t) = (g(t), b(t))^\top$, $t \in [t_0, t_f]$. The system dynamics, which describe the battery/generator system are as follows:

$$\dot{g}(t) = -u(t)\dot{m}(u(t))\bar{G} \tag{13}$$

$$\dot{b}(t) = \frac{P(u(t), T(t))}{V(b(t), P(u(t), T(t)))C_m} \tag{14}$$

Here, $\dot{m}(\cdot)$ is a nonlinear function to give normalized fuel-burn rate as a function of the generator throttle $u(t)$, and $\bar{G}$ is a conversion term to convert the $u(t)\dot{m}(u(t))$ term to fuel burn rate. Further, $P(\cdot)$ gives the net power draw on the battery dependent on generator throttle, $u(t)$, and UAV motor load, $T(t)$. Further, $V(\cdot)$ is a nonlinear function for calculating battery voltage and is a function of the control and net power draw on the battery. Here, we consider the voltage to change with power load applied and current battery state-of-charge (SOC). The model we implement here for simulations is the ohmic-drop model from Hu et al. (2012), and the model for $\dot{m}(\cdot)$ is from Sarkan et al. (2019). Both are normalized to be used in our OCP. The value C_m is the battery maximum SOC in Coloumbs so that SOC in Coloumbs can be converted to an SOC $\in [0, 1]$. The control of the system is the throttle of the generator and is defined as $u(t) \geq 0 \ \forall t \in [t_0, t_f]$. The throttle $u(t)$ is a non-negative function so that the fuel level of the aircraft is away monotonically decreasing as the dynamics in (13) suggest. The control and states are constrained as follows:

$$0 \leq u(t) \leq Q(x(t), y(t), z(t)) \qquad \forall t \in [t_0, t_f] \tag{15}$$

$$B_{\min} \leq b(t) \leq B_{\max} \qquad \forall t \in [t_0, t_f] \tag{16}$$

$$0 \leq g(t) \qquad \forall t \in [t_0, t_f] \tag{17}$$

$$b(t_f) \geq B_f \tag{18}$$

The constraints restrict the operation of the generator in regions deemed as quiet zones. $Q(x(t), y(t), z(t))$ defines the regions of space where the platform can run the generator or not. $B_{\min}$ is the minimum acceptable battery state. $B_{\max}$ is the maximum acceptable battery state. The fuel level $g(t)$. The minimum battery charge required at the terminal location is given by B_f.

The objective, as described, is to minimize fuel consumption – that is, we desire path plans which leverage the battery/generator system so as to minimize the fuel usage.

$$\min_{u(t)} J(\mathbf{x}(t), u(t), t) = \bar{G} \int_{t_0}^{t_f} u(t)\ \mathrm{d}t \tag{19}$$

Equivalently, the objective of minimizing fuel usage over the entire time as described in (19) may be formulated as the maximizing the fuel level at final time or simply:

$$J(\mathbf{x}(t), u(t), t) = -g(t_f) \tag{20}$$

Due to the numerical complexity of solving this optimal control problem, the direct optimal control approach of even collocation is selected Betts (1998, 2010); Kelly (2017); Kirk (2004). While the pseudospectral method and adaptive meshing techniques serve as better surrogates for performing direct optimal control; the task of implementing such a method is left for future work. This paper uses evenly-spaced collocation points and directly transcribes the dynamics using the first-order Euler approximation. This approach is very simple and for the sake of obtianing waypoints is of suitable fidelity. The approach is summarized in Fig. 1. The even collocation method (ECM) is less sensitive to initial guess than shooting methods, it doesn't require the usage of an ordinary differential equation (ODE) solver and in-general converges faster than shooting methods Kelly (2017); Betts (1998, 2010). The engineering trade-off is that the dynamics are not as accurately preserved in ECM as provided in shooting, the equations of motion forward in time using an ODE solver; but, the added performance and reduced sensitivity to the initial guesses are considerable advantages.

The even collocation method transcribes the dynamics to a set of N collocation points evenly spaced in time. The dynamics equations in (13) and (14) (represented as $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}(t), u(t), t)$) are those transcribed to a set of equality constraints:

$$\mathbf{h}_k = \mathbf{x}_{k+1} - \mathbf{x}_k - \mathbf{f}(\mathbf{x}_k, u_k, k)\Delta t,\ k = 1...N \tag{21}$$

The objective cost is evaluated for the current guess of control and state and a check if the states satisfy the dynamics, the objective is a local minimum, and the various path constraints and equality constraints are satisfied. If the constraints are satisfied and the objective is a local minimum then the Karush-Kuhn-Tucker (KKT) conditions are said to be satisfied and the optimal state and control is produced. If, however, the constraints are not satisfied or the objective is not at a local minimum, a nonlinear pro-

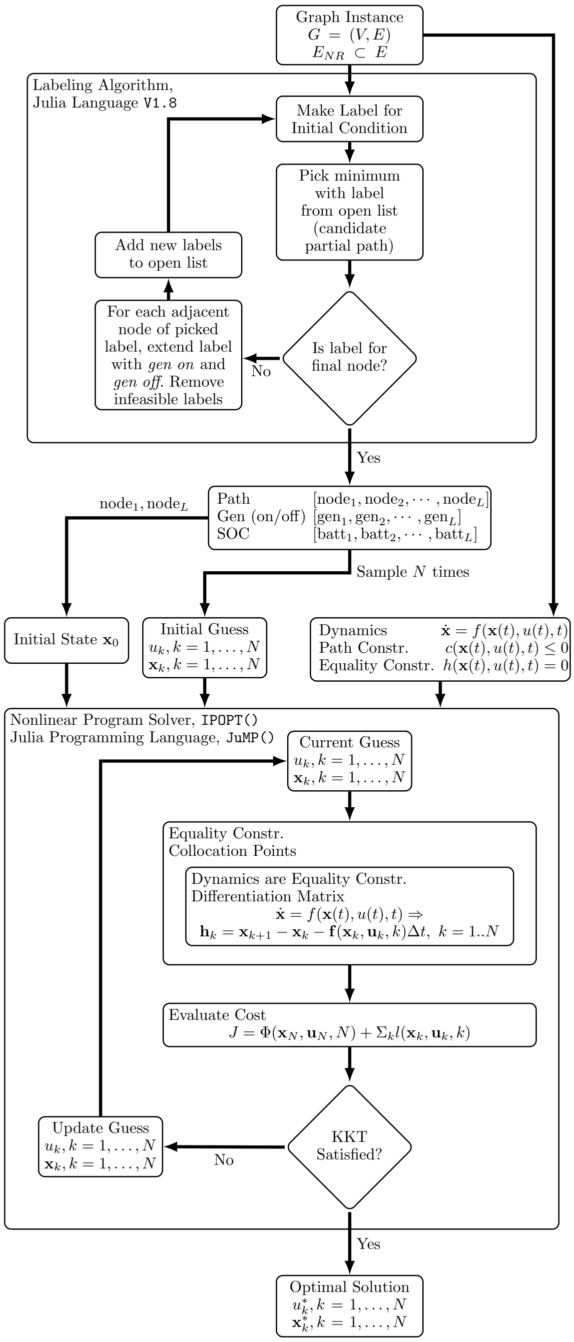

Fig. 1. Hybrid path planning and resource management algorithm

gram solver (in this case the interior point method IPOPT) is used to update the guess for the states and the control Wächter and Biegler (2006). The nonlinear program solver IPOPT is called using the package, "JuMP" as provided by Dunning et al. (2017), a mathematical programming package for Julia language. JuMP, in combination with the Ipopt.jl julia package allows an easy means of posing the direct optimal control problem in the Julia Language and provides an interface to `IPOPT()`.

For a linear system one could use dynamic programming to obtain the solution in a numerical way. We advocate for using a nonlinear program solver to provide optimal strategies when the battery models and generator system

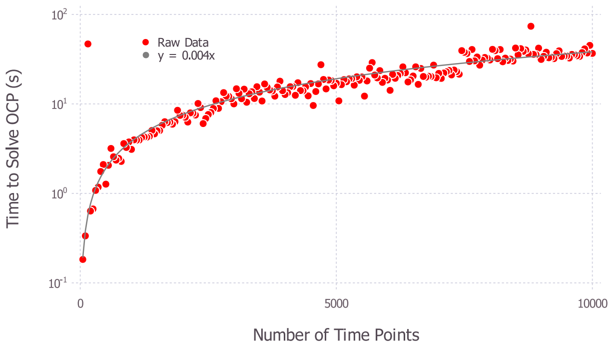

Fig. 2. Time to Solve vs Number of Time Points

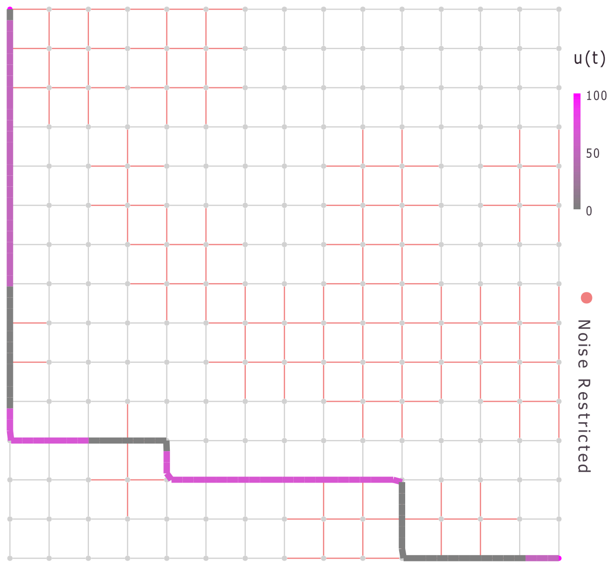

Fig. 3. Vehicle Trajectory

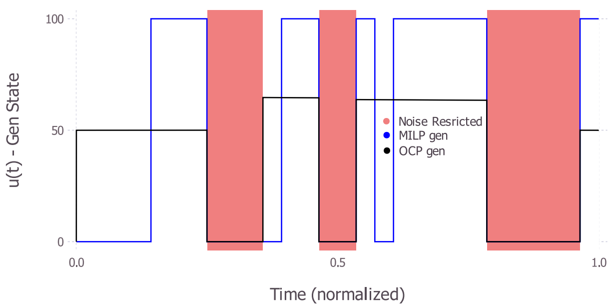

Fig. 4. Generator State and Noise Restrictions

are nonlinear. Furthermore, by solving for the optimal control using optimal control theory, the admissible throttle of the generator is on a continuum rather than on/off – this is more realistic and allows generator policies that aren't possible by only using MILP, seen in results.

5. SIMULATION RESULTS

All results presented here were obtained on a machine running Windows 10 and an Intel i5-4670k processor. Julia v1.8.5 was used, and IPOPT v3.14 was used to solve the nonlinear programming problem, implementing even-

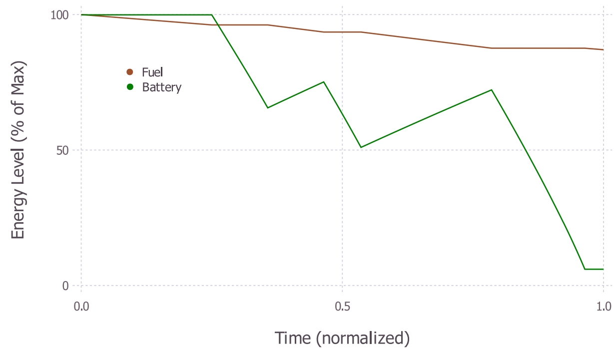

Fig. 5. Example Solution - Battery and Fuel Levels

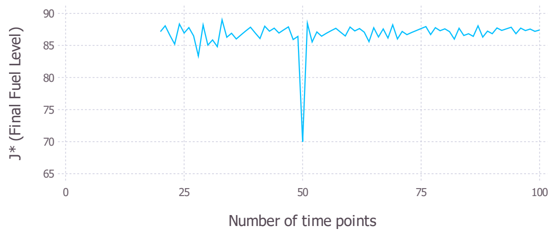

Fig. 6. OCP Cost Function vs Number of Time Points

collocation. The results of the MILP and the results of the OCP are shown here. For tests shown here, a single graph problem and solution is then formulated as an OCP, to the end of minimal fuel usage, with a varying number of time points. A single MILP solution was used repeatedly to solve as an OCP, where only the number of time discretizations within the OCP were changed. The time-to-solve the OCP for varying time discretizations is shown in Fig. 2. The graph instance that is used to formulate the OCP is show in Fig. 3 along with an one OCP generator policy solution. The color along the path indicates generator throttle between 0 and 1. The battery and fuel levels over time, from the same OCP solution as the prior figure, is given in Fig. 5.

Of concern is quality of the generator policy between the MILP solution and the OCP solution. Fig. 4 shows both the MILP generator policy, used as the intial guess in the OCP, and the one returned from the OCP. The MILP, being a discrete formulation, can only consider a finite number of generator control states. As the number of possible generator states increases, so does the branching factor, which has a significant effect on the time complexity of the problem. To limit this, the formulation and algorithm presented in Scott et al. (2022) considers only on and off states. It can be seen clearly in the figure that the optimal control solution runs the generator at varying rates to the end of higher efficiency. Higher generator throttle states increase fuel-burn rate relative to charge provided to the battery. This effect is seen in the OCP solution, where prior to the first noise restricted zone, the generator runs at 50%, however, the generator must run at higher throttle before the second and third noise restricted zones to ensure the battery is charged to be able to fly through these zones on battery-power alone. Thus, the generator is able to run in a higher-efficiency state (in terms of fuel burn with respect to charge to battery) for longer periods of time, as opposed to the "bursts" of maximum throttle that occur in the MILP, which will result in a lower efficiency in terms of fuel burn with respect to charge to battery.

A plot of the optimal objective costs for the OCPs with varying number of time discretization is given in Fig. 6. Objective quality changes very little as time points increase. Between and within noise-restricted zones, the problem constraints do not change. Only at the edges of switches between noise-restrictions does an increase in time resolution offer an advantage, where the higher resolution allows the switching between generator throttles to occur closer to the true-optimal point. This would indicate that pseudo-spectral methods, where the nodes of time-discretizations are allowed to move, would offer an advantage, and likely requiring a smaller number of time points for the same solution quality.

6. CONCLUSIONS AND FUTURE WORK

This paper formulates the problem of controlling the fuel source for a hybrid UAV for a provided path with noise restrictions in the framework of optimal control problem. This work serves as an initial study for a larger problem where path planning and fuel decision are combined. The noise-restrictions and energy constraints couple the energy management and path planning problems. The work presented here is intended to be extended to finding the optimal path and generator policy in tandem as an OCP, likely requiring more advanced solver techniques.

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